Barker College

Student Number: ……………………………………

2015

TRIAL
HIGHER SCHOOL CERTIFICATE

Mathematics Extension 1

AM Friday 7th August

Staff Involved:

- VAB*
- BJB*
- MRB
- KJL
- GIC
- WMD
- DZP

90 copies

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using blue or black pen. Black pen is preferred
- Board-approved calculators may be used.
- Make sure your Barker Student Number is on each page of your solutions.
- On the back of this page is the answer sheet for Section I.
- A table of standard integrals is provided at the back of this paper.
- In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

Total marks – 70

Section I **Pages 2 - 5**

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II **Pages 6 – 12**

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section.

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ANSWER SHEET

Section I – Multiple Choice

Detach this page and hand it in separately.

Choose the best response and clearly shade the corresponding circle.

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

Student Number …......................................

2015 Barker College
Trial HSC Extension 1 Mathematics examination

Section I – Multiple Choice (10 marks) Attempt Questions 1–10

Use the multiple choice answer sheet provided.

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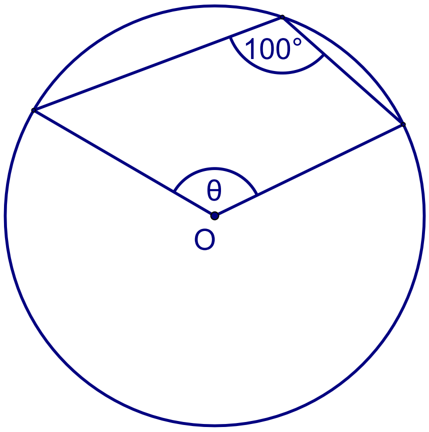
1. O is the centre of the circle. Find the value of θ



NOT TO SCALE

(A) $\theta = 80^\circ$ (B) $\theta = 160^\circ$

(C) $\theta = 200^\circ$ (D) $\theta = 260^\circ$

2. $\int \frac{dx}{4x^2+16}$ equals which of the following expressions?

(A) $\frac{1}{8}\tan^{-1}\frac{x}{2} + C$ (B) $\frac{1}{4}\tan^{-1}\frac{x}{2} + C$

(C) $\frac{1}{2}\tan^{-1}(2\text{x}) + C$ (D) $\frac{1}{4}\tan^{-1}(2\text{x}) + C$

3. Choose the expression which is equivalent to $\sqrt{\cos^2 4\theta - \sin^2 4\theta}$

(A) $\cos 2\theta - \sin 2\theta$ (B) $\cos 4\theta - \sin 4\theta$

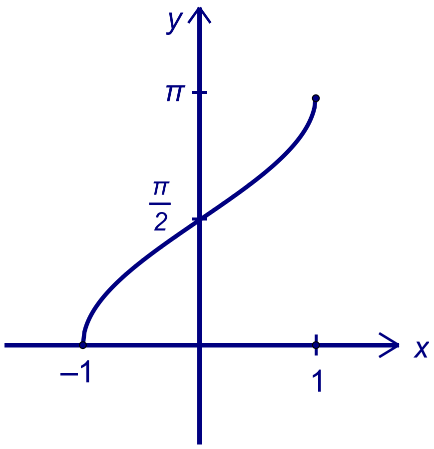
(C) $\sqrt{\cos 8\theta}$ (D) $\sqrt{1 - 2\sin^2 2\theta}$

4. Choose the expression that is equivalent to $3\sin x + \sqrt{3}\cos x$

(A) $2\sqrt{3}\sin\left(x+\frac{\pi}{3}\right)$ (B) $2\sqrt{3}\sin\left(x+\frac{\pi}{6}\right)$

(C) $2\sqrt{3}\sin\left(x-\frac{\pi}{3}\right)$ (D) $2\sqrt{3}\sin\left(x-\frac{\pi}{6}\right)$

5.

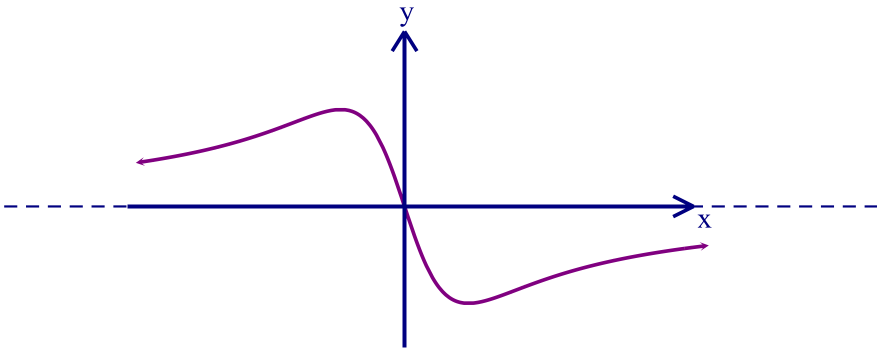


Which combination of the two equations $y = -\cos^{-1} x$ and $y = \frac{\pi}{2} + \sin^{-1} x$ describe the curve shown above?

(A) Both the equations (B) The first equation only

(C) The second equation only (D) Neither of the equations

6. One of the four equations below describes this graph. Choose the correct equation.

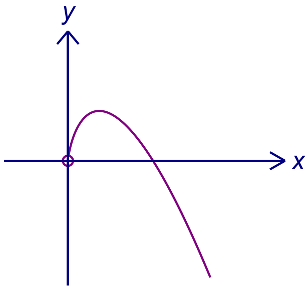


(A) $y = \frac{1}{1+x^2}$ (B) $y = 1 - \frac{1}{1+x^2}$

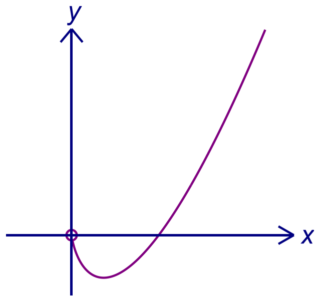
(C) $y = \frac{x}{1+x^2}$ (D) $y = \frac{-x}{1+x^2}$

7. Select the graph which could represent the function $y = x\log_e x$

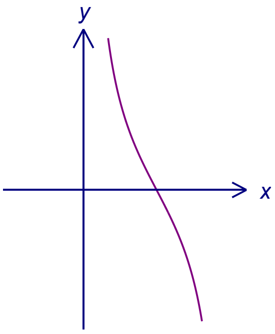
(A)



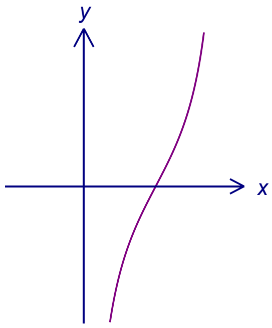
(B)



(C)



(D)



8. Three of these statements are always true and one is sometimes false.

Select the statement which is sometimes false:

(A) $\int f(x)dx = \int f(y)dy$

(B) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

(C) $x - \int f(x)dx = \int (1 - f(x))dx$

(D) $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$

9. What is the general solution to the equation $\sin 2x = 1$, where n is any integer?

(A) $x = \frac{\pm\pi}{4} + n\pi$

(B) $x = \frac{\pm\pi}{4} + 2n\pi$

(C) $x = \frac{\pi}{4} + n\pi$

(D) $x = \frac{\pi}{4} + 2n\pi$

10. For the function $y = \sin^{-1}\left(1 - x^2\right)$ what is the natural domain?

(A) $-\sqrt{2} \le x \le \sqrt{2}$

(B) $-1 \le x \le 1$

(C) $0 \le x \le \sqrt{2}$

(D) $0 \le x \le 1$

End of Section 1

Section II

60 marks
Attempt Questions 11 – 14
Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet.
Extra writing booklets are available.

In Questions 11 – 14, show relevant mathematical reasoning and/or calculations

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Question 11 (15 marks) **[Use a SEPARATE writing booklet]** **Marks**

(a) Solve $\dfrac{x^2 + 20}{x - 4} < -4$ **3**

(b) Find

(i) $\displaystyle\int \frac{1}{\sqrt{\frac{1}{9} - x^2}}\, dx$ **1**

(ii) $\displaystyle\int \sin^2 x \, dx$ **2**

Question 11 continues on page 7

Question 11 (continued) **Marks**

(c) A population of birds on an island changes according to the equation $\dfrac{dP}{dt} = k(P-3500)$

Initially there were 40 individuals and after 5 years the population had increased to 125.

.

(i) Verify that $P = 3500 - Ae^{kt}$ satisfies the above equation, where P is the population, t is the time in years and A and k are constants. **1**

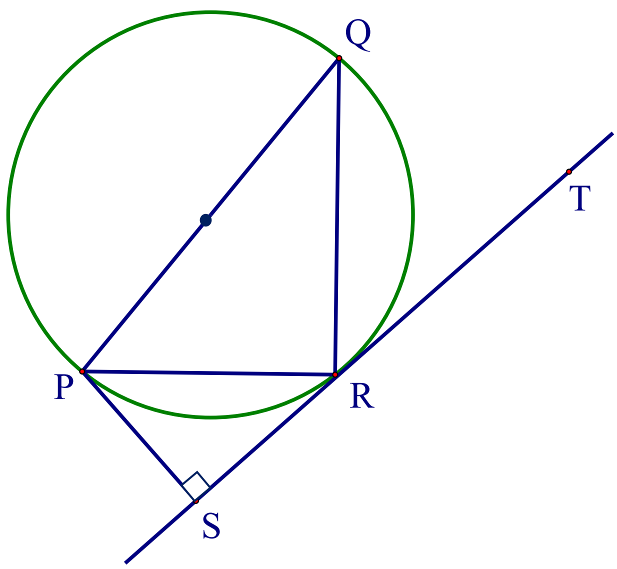
(ii) Find the value of A and find the value of k to 3 significant figures. **3**

(iii) Sketch the function $P = 3500 - Ae^{kt}$ for $t \geq 0$ showing any intercepts and asymptotes. **2**

(d) In the diagram PQ is a diameter of the circle. TS is a tangent to the circle at R and PS is the interval from P perpendicular to TS.

Copy or trace the diagram into your examination booklet.

Prove that PR bisects $\angle QPS$ **3**

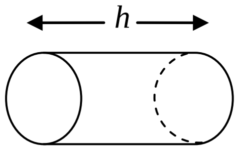


NOT TO SCALE

End of Question 11

Question 12 (15 marks) [Use a SEPARATE writing booklet] **Marks**

(a) A sculptor is rolling modelling clay on a board.
The clay is in the shape of a cylinder and has constant volume 20π cm^3.
As she rolls it into a thinner cylinder, the radius decreases at a constant rate of 2 cm per minute.
Find the rate of increase of the height h of the cylinder with respect to time, at the moment when the radius is 4 cm. **3**

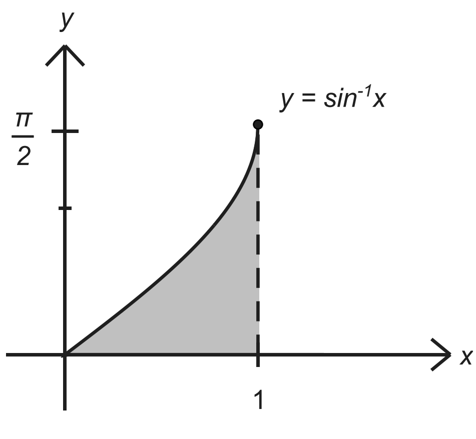


(b) Prove by mathematical induction that for all positive integers n

$$2\times1! + 5\times2! + 10\times3! + ... + (n^2+1)n! = n\times(n+1)!$$ **4**

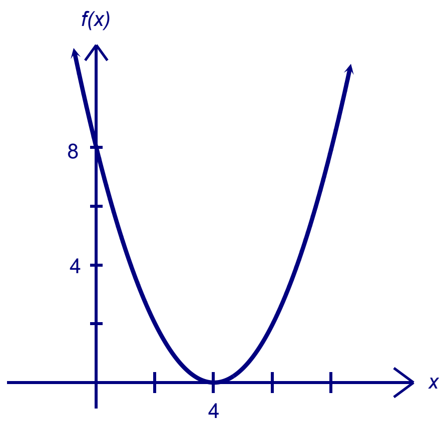
(c) (i) Show that the derivative of $x\sin^{-1}x + \sqrt{1-x^2}$ is $\sin^{-1}x$ **2**

(ii) Hence, determine the shaded area shown which is bounded by the curve $y=\sin^{-1}x$, the x-axis and the line $x=1$ **2**



(d) The function $f(x)=\frac{1}{2}(x-4)^2$ is shown.

$g(x)=f(x)$ over the limited domain $x\geq4$



(i) Find $g^{-1}(x)$, the inverse function of $g(x)$ **2**

(ii) Find the point of intersection of $g(x)$ and $g^{-1}(x)$ **2**

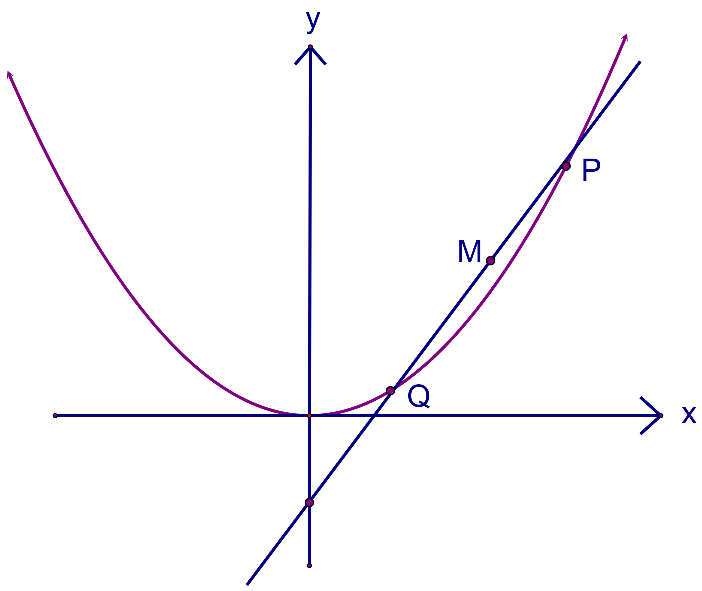
End of Question 12

Question 13 (15 marks) **Use a SEPARATE writing booklet]** **Marks**

(a) A class consists of 12 girls and 10 boys.

How many selections of 4 students are there which contain at least 2 girls? **2**

(b)



$P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are two points on the parabola $x^2 = 4ay$

(i) Write down the gradient of the chord PQ. **1**

(ii) Hence show the equation of the chord PQ is $y - \frac{1}{2}(p+q)x + apq = 0$ **2**

(iii) P and Q move so that the chord PQ passes through the point $(0, -a)$.

Show that $pq = 1$. **1**

(iv) Given that M is the midpoint of PQ with coordinates

$\left[a(p+q)\ ,\ \frac{1}{2}a(p^2+q^2)\right]$ (DO NOT PROVE THIS),

find the locus of M. **2**

Question 13 continues on page 10

(c) A particle moves according to the equation $v^2 = 16 - 4x^2$ where x is its position in metres and v is its velocity in metres per second.

Initially the particle is at the origin and moves off in a positive direction.

(i) Prove the particle has acceleration in the form $a = -n^2 x$ where n is a constant. **1**

(ii) Find the amplitude of the motion, explaining how this is found. **2**

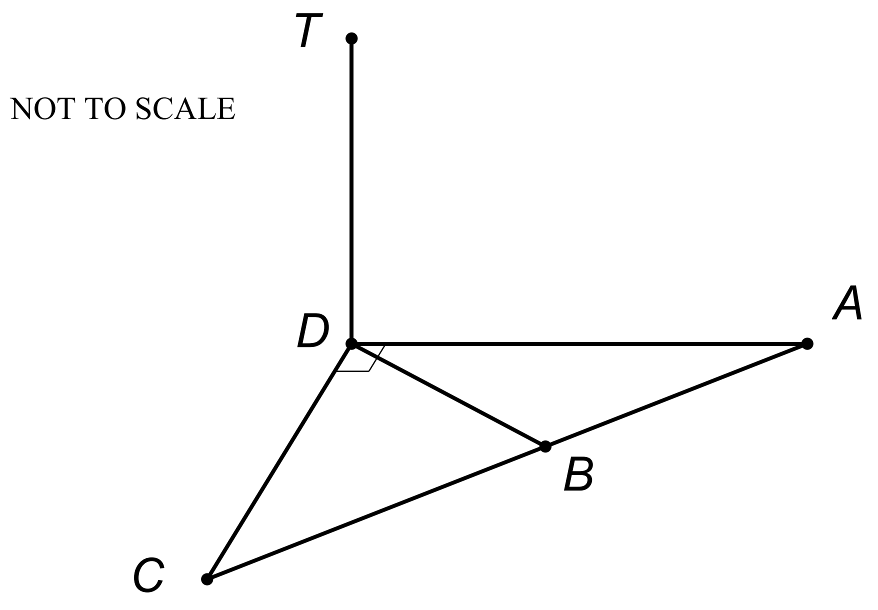
(iii) Use integration to find an expression for x as a function of time t, beginning with $v = \sqrt{16 - 4x^2}$ **4**

End of Question 13

Question 14 (15 marks) **[Use a SEPARATE writing booklet]** **Marks**

(a) Three surveyors A, B and C are observing a tower of height h situated on flat ground. A is due east of the tower, C is due south of it and B is somewhere between A and C on the line AC. The tower's base is at D and its top is at T.

The angles of elevation of the top of the tower from A, B and C are $17^o, 20^o$ and 24^o respectively. Note that $\angle ABD$ is obtuse.



(i) Show $AD = h\tan 73^\circ$ and write similar expressions for BD and CD. **1**

(ii) Find $\angle DAC$ to the nearest minute. **1**

(iii) Hence, find the bearing of B from the base of the tower D. **3**

Question 14 continues on page 12

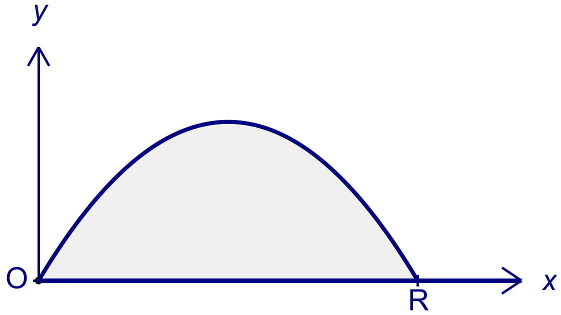
Question 14 (continued) **Marks**

(b) A projectile is fired at V m/s at an angle θ with the horizontal.

The equations of motion of the particle are

$x = Vt\cos\theta, \qquad y = -\frac{1}{2}gt^2 + Vt\sin\theta$

DO NOT PROVE THIS



(i) Using the above equations, show the Cartesian equation of motion of the particle is $y = \frac{-g}{2V^2\cos^2\theta}x^2 + x\tan\theta$ **1**

(ii) The range of the projectile is given by

$$R = \frac{2V^2\sin\theta\cos\theta}{g} \qquad \text{(DO NOT PROVE THIS)}$$

Show that the shaded area A under the path of the parabola, for fixed values of V and θ, is $A = \frac{2V^4}{3g^2}\sin^3\theta\cos\theta$ **3**

(iii) The shaded area shown is to be illuminated by rockets fired from O, fitted with downward-directed lights.

If θ is allowed to vary while V is kept constant, there is a maximum value of A for a value of θ between 0 and $\frac{\pi}{2}$.

Find this value of θ. **2**

(c) (i) Prove that $\cos h - 1 = -2\sin^2\frac{h}{2}$ **1**

(ii) Hence, given that $f(x) = \cos x$, find $f'(x)$ from first principles,

using the definition $f'(x) = \lim_{h\to 0}\left[\frac{f(x+h) - f(x)}{h}\right]$ **3**

End of Paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \quad \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Trial HSC Ext 1 SOLUTIONS 2015

1 Reflex angle at O = 200°

$\therefore \theta = 360^\circ - 200^\circ = 160^\circ$

B

2 $\displaystyle\int \frac{1}{4x^2+16}\,dx = \frac{1}{4}\int \frac{1}{x^2+4}\,dx$

$= \frac{1}{4} \times \frac{1}{2} \tan^{-1}\frac{x}{2} + c$

A

3 $\cos^2 x - \sin^2 x = \cos 2x$

$\therefore \sqrt{\cos^2 4\theta - \sin^2 4\theta}$

$= \sqrt{\cos 8\theta}$

C

4 B

Let $3\sin x + \sqrt{3}\cos x$

$\equiv R\sin(x+\alpha)$

$\equiv R\sin x\cos\alpha + R\cos x\sin\alpha$

Equate coefficients

$\therefore 3 = R\cos\alpha$ —(1), $\sqrt{3} = R\sin\alpha$ —(2)

$\frac{(2)}{(1)}$ $\therefore \frac{R\sin\alpha}{R\cos\alpha} = \frac{\sqrt{3}}{3}$

$\therefore \tan\alpha = \frac{1}{\sqrt{3}}$, $\therefore \alpha = \frac{\pi}{6}$

$(1)^2$: $9 = R^2\cos^2\alpha$ —(3)

$(2)^2$: $3 = R^2\sin^2\alpha$ —(4)

(3) + (4) $12 = R^2(\cos^2\alpha + \sin^2\alpha)$

$\therefore 12 = R^2$

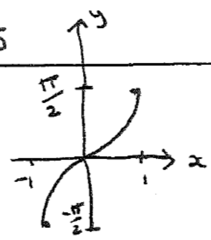
$\therefore R = \sqrt{12} = 2\sqrt{3}$

$\therefore 3\sin x + \sqrt{3}\cos x$

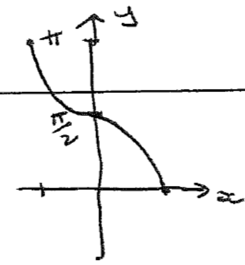
$= 2\sqrt{3}\sin\left(x + \frac{\pi}{6}\right)$

5 C

2nd equation only



$y = \sin^{-1}x$



$y = \cos^{-1}x$

shift up $\frac{\pi}{2}$ to get $y = \frac{\pi}{2} + \sin^{-1}x$

✓ Yes, matches graph.

But $y = -\cos^{-1}x$ is reflected in x-axis, not y-axis

✗ No, does not match graph.

6 A does not pass through (0, 0) ×

B is even, not odd ×

C is in Quadrants 1 and 3 ×

D ✓

7 B

for $0 < x < 1$, $y < 0$

for $x > 1$, $y > 0$

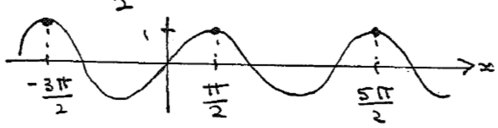
and as $x \to 0$, $y \to 0$

8 D

D is only true for even functions and is false for other functions

9 $\sin 2x = 1$ C

$\therefore 2x = \frac{\pi}{2} + 2n\pi$



$\therefore x = \frac{\pi}{4} + n\pi$

10 $y = \sin^{-1}(1-x^2)$ A

Domain of $\sin^{-1}m$ is $-1 \le m \le 1$

$\therefore -1 \le 1 - x^2 \le 1$

$\therefore -2 \le -x^2 \le 0$

$\therefore 0 \le x^2 \le 2$

$\therefore -\sqrt{2} \le x \le \sqrt{2}$

1.	B	6.	D
2.	A	7.	B
3.	C	8.	D
4.	B	9.	C
5.	C	10.	A

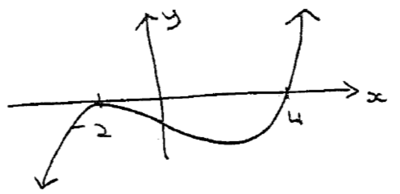
11 a) $\frac{x^2+20}{x-4} < -4$

$\therefore \frac{x^2+20}{x-4} + 4 < 0$

$\therefore \frac{x^2+20+4x-16}{x-4} < 0$

$\therefore \frac{x^2+4x+4}{x-4} < 0$

$\therefore (x+2)^2(x-4) < 0$



$\therefore x < -2, \quad -2 < x < 4$

b) i) $\int \frac{1}{\sqrt{\frac{1}{9}-x^2}}\,dx = \sin^{-1}\frac{x}{\frac{1}{3}} + c$

$= \sin^{-1}3x + c$

b ii) $\int \sin^2 x\,dx = \int \frac{1}{2}(1-\cos 2x)\,dx$

$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$

$= \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

c) i) For $P = 3500 - Ae^{kt}$

$\therefore \frac{dP}{dt} = -Ake^{kt}$

and $k(P-3500)$

$= k(3500 - Ae^{kt} - 3500)$

$= -kAe^{kt}$

$\therefore \frac{dP}{dt} = k(P-3500)$ as required

$\therefore$ it is a solution

11c ii) $\left.\begin{matrix} t=0 \\ P=40 \end{matrix}\right\} \Rightarrow P = 3500 - Ae^{kt}$

$\therefore 40 = 3500 - A$

$\therefore A = 3460$

$\left.\begin{matrix} t=5 \\ P=125 \end{matrix}\right\} \Rightarrow P = 3500 - 3460e^{kt}$

$\therefore 125 = 3500 - 3460e^{5k}$

$\therefore 3460e^{5k} = 3375$

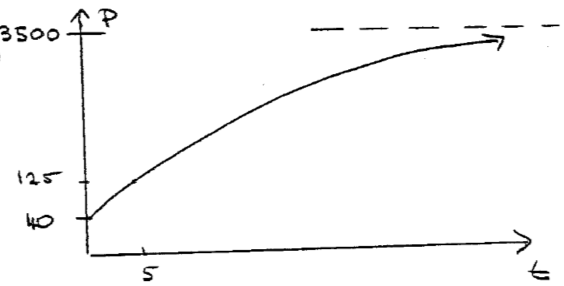
$\therefore e^{5k} = \frac{3375}{3460} = 0.97543\ldots$

$\therefore 5k = \log_e 0.97543\ldots$

$\therefore k = \frac{1}{5}\log_e 0.97543\ldots$

$\therefore k = -0.00497$ (3 s f)

11c iii)



d)

$\angle PRQ = 90^\circ$ ($\angle$ in a semicircle)

$\angle PRS = \angle PQR$ ($\angle$ between chord and tangent = $\angle$ in alternate segment)

$\angle RPS = 90^\circ - \angle PRS$ ($\angle$ sum ΔRPS)

$\angle QPR = 180^\circ - \angle PRQ - \angle PQR$ ($\angle$ sum ΔPQR)

$= 180^\circ - 90^\circ - \angle PRS$ (proved above)

$= 90^\circ - \angle PRS$

$\therefore \angle RPS = \angle QPR$ (both equal $90^\circ - \angle PRS$)

$\therefore$ RP bisects $\angle QPS$

12a) $20\pi = \pi r^2 h$

$\therefore 20 = r^2 h$

$\therefore h = 20r^{-2}$

$\therefore \frac{dh}{dr} = -40r^{-3} = \frac{-40}{r^3}$

Find $\frac{dh}{dt} = ?$

$\frac{dh}{dt} = \frac{dh}{dr} \times \frac{dr}{dt}$

$= \frac{-40}{r^3} \times -2$

$= \frac{80}{r^3}$

$\therefore$ when $r = 4$

$\frac{dh}{dt} = \frac{80}{4^3} = 1.25$ cm/min

12c) i) Let $y = x\sin^{-1}x + \sqrt{1-x^2}$

$\therefore y = x\sin^{-1}x + (1-x^2)^{\frac{1}{2}}$

$\therefore \frac{dy}{dx} = \sin^{-1}x + \frac{x}{\sqrt{1-x^2}} + \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \times -2x$

$= \sin^{-1}x + \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$

$= \sin^{-1}x$

c) ii) Area $= \int_0^1 \sin^{-1}x \, dx$

$= \left[x\sin^{-1}x + \sqrt{1-x^2}\right]_0^1$

$= 1\sin^{-1}1 + 0 - 0 - 1$

$= \left(\frac{\pi}{2} - 1\right)$ un^2

12b) When $n=1$, LHS $= 2\times 1! = 2$, RHS $= 1\times 2! = 2 =$ LHS $\therefore$ true for $n=1$

Assume true some integer $n=k$

$\therefore$ Assume $2\times1! + 5\times2! + \ldots + (k^2+1)k! = k(k+1)!$

Aim: Prove $2\times1! + 5\times2! + \ldots + (k^2+1)k! + [(k+1)^2+1](k+1)! = (k+1)(k+2)!$

Proof $2\times1! + 5\times2! + \ldots + (k^2+1)k! + [(k+1)^2+1](k+1)!$

$= k(k+1)! + (k^2+2k+2)(k+1)!$ (by assumption)

$= (k+1)!(k^2+3k+2)$

$= (k+1)!(k+2)(k+1) = (k+2)!(k+1)$

$\therefore$ If true for $n=k$ proved true for $n=k+1$ $\therefore$ By math ind'n true for all positive integers n

12d i)

$\therefore f(x)$ is $y = \frac{1}{2}(x-4)^2$

$\therefore f^{-1}(x)$ is $x = \frac{1}{2}(y-4)^2$

$\therefore 2x = (y-4)^2$

$\therefore \pm\sqrt{2x} = y-4$

$\therefore y = 4 \pm \sqrt{2x}$

$\therefore y = 4 + \sqrt{2x}$

[Since domain of $f(x)$ is $x \geq 4$ so range of $f^{-1}(x)$ is $y \geq 4$]

12d.ii) $f(x)$ and $f^{-1}(x)$ intersect on line $y=x$

$\therefore$ Solve $f(x) = x$

$\therefore \frac{1}{2}(x-4)^2 = x$

$\therefore x^2 - 8x + 16 = 2x$

$\therefore x^2 - 10x + 16 = 0$

$\therefore (x-8)(x-2) = 0$

$\therefore x = 2$ or 8

$\therefore$ Pt $= (2, 2)$ or $(8, 8)$

But $y > 4$

$\therefore$ Pt is $(8, 8)$

13a) Selections are 2g2b or 3g1b or 4g

No selections $= {}^{12}C_2 \cdot {}^{10}C_2 + {}^{12}C_3 \cdot {}^{10}C_1 + {}^{12}C_4$

$= 2970 + 2200 + 495$

$= 5665$

13b) i) $m = \frac{p+q}{2}$

ii) $y - ap^2 = \frac{p+q}{2}(x - 2ap)$

$\therefore y - ap^2 = \frac{1}{2}(p+q)x - ap(p+q)$

$\therefore y - ap^2 - \frac{1}{2}(p+q)x + ap(p+q) = 0$

$\therefore y - ap^2 - \frac{1}{2}(p+q)x + ap^2 + apq = 0$

$\therefore y - \frac{1}{2}(p+q)x + apq = 0$

iii) Sub $(0, -a)$ in (ii)

$\therefore -a + apq = 0$

$\therefore apq = a$

$\therefore pq = 1$

iv) <u>P+M</u>

$X = a(p+q)$

$\therefore \frac{X}{a} = p+q$ — ①

$Y = \frac{1}{2}a(p^2+q^2)$

$\therefore 2Y = a(p^2 + 2pq + q^2 - 2pq)$

$= a(p^2 + 2pq + q^2 - 2)$

$\therefore 2Y = a(p+q)^2 - 2a$ — ②

Sub ① in ②

$\therefore 2Y = a \times \frac{X^2}{a^2} - 2a$

$\therefore 2Y = \frac{X^2}{a} - 2a$

$\therefore 2aY = X^2 - 2a^2$

$\therefore X^2 = 2a^2 + 2aY$

or $X^2 = 2a(Y+a)$

13 c) i) $v^2 = 16 - 4x^2$

$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$= \frac{d}{dx}\left(8 - 2x^2\right)$

$= -4x$

$= -2^2 x$

as required

c) ii) when $v = 0$, $16 - 4x^2 = 0$

$\therefore x^2 = 4$

$\therefore x = \pm 2$

These are the two extreme positions of motion.

$\therefore$ Amplitude = 2 metres.

c iii) $v = \sqrt{16 - 4x^2}$

$\therefore \frac{dx}{dt} = \sqrt{16 - 4x^2}$

$\therefore \frac{dt}{dx} = \frac{1}{\sqrt{16-4x^2}} =$

$= \frac{1}{2} \times \frac{1}{\sqrt{4 - x^2}}$

$\therefore t = \frac{1}{2}\sin^{-1}\left(\frac{x}{2}\right) + c$

When $t = 0$, $x = 0$ $\Big\}$ $\therefore 0 = 0 + c$ $\therefore c = 0$

$\therefore t = \frac{1}{2}\sin^{-1}\left(\frac{x}{2}\right)$

$\therefore 2t = \sin^{-1}\left(\frac{x}{2}\right)$

$\therefore \frac{x}{2} = \sin(2t)$

$\therefore x = 2\sin(2t)$

14 a) i) In ΔADT $\tan 73° = \frac{AD}{h}$

$\therefore AD = h \tan 73°$

$BD = h \tan 70°$

$CD = h \tan 66°$

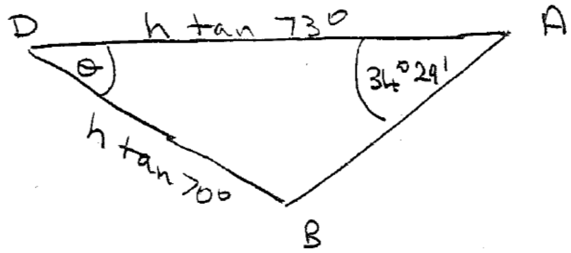
a ii) In ΔACD

$\tan \angle DAC = \frac{CD}{AD} = \frac{h \tan 66°}{h \tan 73°}$

$= 0.68668\ldots$

$\therefore \angle DAC = 34° 29'$

a iii)



$\frac{\sin B}{h \tan 73°} = \frac{\sin 34°29'}{h \tan 70°}$

$\therefore \sin B = \frac{h \tan 73° \sin 34°29'}{h \tan 70°}$

$= 0.674\ldots$

$\therefore \angle B = \angle ABD = 42°23'$ or $137° 37'$

But $\angle ABD$ is obtuse (data)

$\therefore \angle ABD = 137° 37'$

$\therefore \angle ADB = 180° - 137°37 - 34°29'$

$= 7° 54'$

$\therefore$ Bearing of B from D is $97° 54'$

14b)

i) $x = Vt\cos\theta$

$\therefore t = \dfrac{x}{V\cos\theta}$ — ①

sub in ② $\quad y = -\frac{1}{2}gt^2 + Vt\sin\theta$ — ②

$$\therefore y = \frac{-g}{2} \times \frac{x^2}{V^2\cos^2\theta} + \frac{V\sin\theta \cdot x}{V\cos\theta}$$

$$\therefore y = \frac{-g}{2V^2\cos^2\theta}x^2 + x\tan\theta$$

ii)
$$\text{Area} = \int_0^R \frac{-g}{2V^2\cos^2\theta}x^2 + x\tan\theta \, dx$$

$$= \left[\frac{-gx^3}{6V^2\cos^2\theta} + \frac{x^2\tan\theta}{2}\right]_0^R$$

$$= \frac{-gR^3}{6V^2\cos^2\theta} + \frac{R^2\tan\theta}{2}$$

$$= \frac{-g}{6V^2\cos^2\theta} \times \frac{8V^6\sin^3\theta\cos^3\theta}{g^3} + \frac{\tan\theta}{2} \times \frac{4V^4\sin^2\theta\cos^2\theta}{g^2}$$

$$= \frac{-4V^4\sin^3\theta\cos\theta}{3g^2} + \frac{2V^4\sin^3\theta\cos\theta}{g^2}$$

$$= \frac{-4V^4\sin^3\theta\cos\theta + 6V^4\sin^3\theta\cos\theta}{3g^2}$$

$$\therefore \text{Area} = \frac{2V^4\sin^3\theta\cos\theta}{3g^2}$$

14 b) iii) $A = \frac{2v^4}{3g^2} \sin^3\theta \cos\theta$

$\therefore \frac{dA}{d\theta} = \frac{2v^4}{3g^2}\left(3\sin^2\theta\cos^2\theta + - \sin^4\theta\right) = 0$ st pt

$\therefore \sin^2\theta(3\cos^2\theta - \sin^2\theta) = 0$

$\therefore 3\cos^2\theta = \sin^2\theta$ (θ is acute)

$\therefore 3 = \tan^2\theta$

$\therefore \tan\theta = \sqrt{3}$ (θ acute)

$\therefore \theta = \frac{\pi}{3}$

(14c) i) $\cos h - 1 = 1 - 2\sin^2\frac{h}{2} - 1$

$= -2\sin^2\frac{h}{2}$

ii) $f(x) = \cos x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h\to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h\to 0} \frac{\cos x(\cos h - 1) - \sin x \sin h}{h}$$

$$= \lim_{h\to 0} \frac{\cos x(\cos h - 1)}{h} - \lim_{h\to 0} \frac{\sin x \sin h}{h}$$

$$= \cos x \lim_{h\to 0} \frac{\cos h - 1}{h} - \sin x \lim_{h\to 0} \frac{\sin h}{h}$$

$$= \cos x \lim_{h\to 0} \frac{-2\sin^2\frac{h}{2}}{h} - \sin x \times 1$$

$$= -\cos x \lim_{h\to 0} \frac{\sin^2\frac{h}{2}}{\frac{h}{2}} - \sin x$$

$$= -\cos x \lim_{h\to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{\sin\frac{h}{2}}{1} - \sin x$$

$$= -\cos x \times 1 \times 0 - \sin x$$

$$= 0 - \sin x$$

$$= -\sin x$$